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6 THE SIMPLEST 'HUECKEL' EDGE DETECTOR IS A ROBERTS OPERATOR.

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## ABSTRACT

Hueckel's edge detector finds the best-fitting ideal step edge to a given picture neighborhood, by expanding the neighborhood and step edge in terms of a set of nine basis functions. The simplest case of this approach uses a 2-by-2 neighborhood and three basis functions. This case is solved explicitly using elementary methods. The magnitude of the best-fitting step edge for the neighborhood

AB
CD

turns out to be the Roberts operator max (|A-D|,|B-C|).

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## 1. Introduction

Hueckel [1-2] developed an approach to edge detection based on fitting an ideal step edge to a given picture neighborhood. The fitting was done by expanding both the step edge and the neighborhood in terms of a set of orthogonal basis functions, and minimizing the sum of the squared differences between corresponding coefficients. To simplify the computation, the expansion is truncated; Hueckel used a nine-term expansion.

Several simplifications of Hueckel's approach have also been investigated. Nevatia [3] used a subset of Hueckel's basis; O'Gorman [4] used a set of two-dimensional Walsh functions defined on a square; Meró and Vássy [5] used only two basis functions, defined by diagonally subdividing a square, to determine edge orientation; and Hummel [6] used a set of optimal basis functions derived from the Karhunen-Loève expansion of the local image values.

In this correspondence we present an elementary treatment of edge fitting in the simplest possible nontrivial case. We use a 2-by-2 picture neighborhood $\begin{matrix} A & B \\ C & D \end{matrix}$, and a step function $s(x,y)$ passing through the center of this neighborhood (which we take to be the origin), defined by

$$s(x,y) = \begin{cases} a & \text{if } x \sin\theta \geq y \cos\theta \\ b & \text{otherwise} \end{cases}$$

We use only three basis functions, namely

$$\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix} \qquad \begin{matrix} -1 & -1 \\ 1 & 1 \end{matrix} \qquad \begin{matrix} 1 & -1 \\ 1 & -1 \end{matrix}$$

It turns out that the magnitude of the best-fitting step edge derived in this way is just max $(|A-D|, |B-C|)$, the Roberts operator [7]. Thus this correspondence has two purposes: to illustrate how Hueckel-type edge detectors can be derived using elementary methods, and to provide a new motivation for the max version of the Roberts operator.

## 2. Derivation

Let the coefficients of $f(x,y) \equiv \begin{smallmatrix} A & B \\ C & D \end{smallmatrix}$ with respect to these basis functions be $f_0, f_1$, and $f_2$, respectively; then readily we have

$$
\begin{aligned}
f_0 &= (A+B+C+D)/4 \equiv S/4 \\
f_1 &= (-A-B+C+D)/4 \\
f_2 &= (A-B+C-D)/4
\end{aligned}
\tag{1}
$$

Similarly, let the coefficients of $s(x,y)$ be $s_0, s_1$, and $s_2$; then readily

$$
\begin{aligned}
s_0 &= (a+b)/2 \\
s_1 &= \frac{\theta}{2\pi}(b-a) + \frac{\pi-\theta}{2\pi}(a-b) = \frac{2\theta-\pi}{2\pi}(b-a) \text{ if } 0 \le \theta \le \pi \\
&= \frac{\theta-\pi}{2\pi}(a-b) + \frac{2\pi-\theta}{2\pi}(b-a) = \frac{3\pi-2\theta}{2\pi}(b-a) \text{ if } \pi \le \theta \le 2\pi \\
s_2 &= \frac{\theta+\frac{\pi}{2}}{2\pi}(b-a) + \frac{\frac{\pi}{2}-\theta}{2\pi}(a-b) = \frac{\theta}{\pi}(b-a) \text{ if } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \\
&= \frac{\theta-\frac{\pi}{2}}{2\pi}(a-b) + \frac{\frac{3\pi}{2}-\theta}{2\pi}(b-a) = \frac{\pi-\theta}{\pi}(b-a) \text{ if } \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}
\end{aligned}
\tag{2}
$$

We now want to minimize $E^2 \equiv (f_0-s_0)^2 + (f_1-s_1)^2 + (f_2-s_2)^2$; because of the way $s_1$ and $s_2$ are defined, this must be done separately for $\theta$ in each quadrant. Actually, since $a$ and $b$ are interchangeable, by symmetry it suffices to treat the first and second quadrants. In fact, in ( 2 ), if we replace $\theta$ by $\theta+\pi$ and interchange $a$ and $b$ in $\frac{3\pi-2\theta}{2\pi}(b-a)$, we obtain $\frac{2\theta-\pi}{2\pi}(b-a)$; and similarly $\frac{\pi-\theta}{\pi}(b-a)$ yields $\frac{\theta}{\pi}(b-a)$.

In the first quadrant we have

$$E^2 = \left(\frac{S}{4} - \frac{a+b}{2}\right)^2 + \left(\frac{-A-B+C+D}{4} - \frac{2\theta-\pi}{2\pi}(b-a)\right)^2$$

$$+ \left(\frac{A-B+C-D}{4} - \frac{\theta}{\pi}(b-a)\right)^2 \qquad (3)$$

Taking partial derivatives of ( 3 ) with respect to a and b and setting them equal to zero, we obtain

$$-\frac{1}{2}\left[\frac{S}{4} - \frac{a+b}{2}\right] + \frac{2\theta-\pi}{2\pi}\left[\frac{-A-B+C+D}{4} - \frac{2\theta-\pi}{2\pi}(b-a)\right]$$

$$+ \frac{\theta}{\pi}\left[\frac{A-B+C-D}{4} - \frac{\theta}{\pi}(b-a)\right] = 0$$

$$(4)$$

$$-\frac{1}{2}\left[\frac{S}{4} - \frac{a+b}{2}\right] - \frac{2\theta-\pi}{2\pi}\left[\frac{-A-B+C+D}{4} - \frac{2\theta-\pi}{2\pi}(b-a)\right]$$

$$- \frac{\theta}{\pi}\left[\frac{A-B+C-D}{4} - \frac{\theta}{\pi}(b-a)\right] = 0$$

Adding gives immediately $S/4 = (a+b)/2$, or $a+b = S/2$, as in the previous solution. Taking the partial derivative of (3) with respect to $\theta$ and equating it to zero gives

$$\left[\frac{-A-B+C+D}{4} - \frac{2\theta-\pi}{2\pi}(b-a)\right] + \left[\frac{A-B+C-D}{4} - \frac{\theta}{\pi}(b-a)\right] = 0 \qquad (5)$$

or

$$\frac{C-B}{2} = \frac{4\theta-\pi}{2\pi}(b-a)$$

so that $(4\theta-\pi)(b-a)/\pi = C - B$ (6)

Also, substituting $S/4 = (a+b)/2$ and ( 5 ) in either equation of ( 4 ) gives

$$\frac{-A-B+C+D}{4} - \frac{2\theta-\pi}{2\pi}(b-a) = 0 \tag{7}$$

so that we also have

$$\frac{A-B+C-D}{4} - \frac{\theta}{\pi}(b-a) = 0 \tag{8}$$

If $\theta$ is not $0$ or $\pi/2$, and $b \neq a$, we can divide (7) by (8) (or vice versa) to obtain

$$\frac{A-B+C-D}{-A-B+C+D} = \frac{\theta}{\theta-\frac{\pi}{2}} \tag{9}$$

from which we readily have

$$\theta(2A-2D) = \frac{\pi}{2}(A-B+C-D)$$

or (if $A \neq D$)

$$\theta = \frac{\pi}{4} \cdot \frac{A-B+C-D}{A-D} = \frac{\pi}{4}\left[1 - \frac{B-C}{A-D}\right] \tag{10}$$

Combining this with (8) gives $b-a = A-D$, and combining this with $b+a = S/2$ gives

$$\begin{aligned} a &= (-A+B+C+3D)/4 = S/4 - (A-D)/2 \\ b &= (3A+B+C-D)/4 = S/4 + (A-D)/2 \end{aligned} \tag{11}$$

Note that by (7), (8), and the fact that $a+b = S/2$, we actually have $E^2 = 0$ for this solution. It is easily verified that if we assume $\theta = 0$ or $\theta = \frac{\pi}{2}$ in (3), and set the partial derivatives with respect to $a$ and $b$ equal to zero, we obtain special cases of this solution. In fact, for $\theta = 0$, we find that $A+C = B+D$, $b = (A+B)/2$, and $a = (C+D)/2$; while for $\theta = \frac{\pi}{2}$ we get $A+B = C+D$, $b = (A+C)/2$, and $A = (B+D)/2$.

In the second quadrant, analogously, we get

$$\begin{aligned} a &= (A+3B-C+D)/4 = S/4 - (C-B)/2 \\ b &= (A-B+3C+D)/4 = S/4 + (C-B)/2 \end{aligned} \tag{12}$$

and

$$\theta = \frac{\pi}{4}\left[3 + \frac{A-D}{B-C}\right] \tag{13}$$

It can be verified that the first and second quadrant solutions agree if $\theta = \pi/2$. Moreover, note that for (10) to actually lie in the first quadrant we must have

$$-1 \le \frac{B-C}{A-D} \le 1$$

which is evidently equivalent to $|B-C| \le |A-D|$. Similarly, for (13) to actually lie in the second quadrant we must have

$$-1 \le \frac{A-D}{B-C} \le 1$$

which is evidently equivalent to $|A-D| \le |B-C|$. Thus by comparing the magnitudes and signs of $A-D$ and $B-C$ we can choose the appropriate best-fitting step edge for the given neighborhood $\begin{matrix} A B \\ C D \end{matrix}$. Note that if $A-D = B-C$ we have $\theta = 0$ in (10) and $\theta = \pi$ in (13); moreover, in this case (11) and (12) also agree, with $a$ and $b$ interchanged. Similarly, if $A-D = C-B$, we have $\theta = \frac{\pi}{2}$ in both (10) and (13), and here (11) and (12) agree too.

In summary, the "best-fitting" step edge to $\begin{matrix}A B\\C D\end{matrix}$ is found as follows:

If $|B-C| \leq |A-D|$, then $\theta = \frac{\pi}{4}[1 - \frac{B-C}{A-D}]$, and $a,b$ are given by (11)

If $|B-C| \geq |A-D|$, then $\theta = \frac{\pi}{4}[3 + \frac{A-D}{B-C}]$, and $a,b$ are given by (12)

The magnitude $|a-b|$ of the edge is $|A-D|$ in the first case, and $|B-C|$ in the second case; in other words, the magnitude is $\max(|A-D|,|B-C|)$. Note that this is just the magnitude of the Roberts operator, using the max of the absolute differences rather than the square root of the sum of the squares [7]. (The slope $\theta$, on the other hand, is not the arc tangent of the ratio of these differences; but its value is reasonable, e.g., if $\begin{matrix}A B\\C D\end{matrix} = \begin{matrix}1 2\\3 4\end{matrix}$ we get $\theta = \frac{\pi}{6}$.)

## 3. Conclusion

We have presented an elementary derivation of step edge fitting in the simplest nontrivial case: a 2-by-2 neighborhood and three basis functions. It turns out that the magnitude of the best-fitting edge to $\begin{matrix} A & B \\ C & D \end{matrix}$ is $\max(|A-D|, |B-C|)$, which is a commonly used version of the Roberts edge detector; thus our derivation provides a new motivation for that detector.

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